

A note on the structure of the goal-programming model: assessment and evaluation

Ahmed K. Rifai

Northern Illinois University, DeKalb, Illinois, USA

The growing complexity of today's business competition has awakened US firms to the fact that consumers have become quality and cost oriented. This, in turn, means that the survival of any US firm in the national and international markets depends to a great extent on the use of scientific techniques in their decision-making processes. The utilization of scientific techniques requires certain steps to be followed. The most important steps are:

- *Identifying the problem.* This step requires a precise description of the problem and its *symptoms*. In some cases, the problem is clear and can be quantified, and in others it requires much research and discussion among the people involved. However, all these issues must be resolved and clarified by consensus of the individuals responsible for solving the problem.
- *Quantifying the problem.* Having clearly identified the problem, the next step is to describe the problem in a quantitative (mathematical) manner.
- *Solving the problem.* Once the problem is mathematically formulated, the next step is to obtain the solution of the mathematical formulation – that is, to obtain quantifiable numbers for the decision variables. There are many quantitative models available for any decision maker to obtain the best possible solution. Also, there are many “user-friendly” software options which can solve the mathematical model describing the problem. The issue, however, is that it is very important that the structure of the quantitative model is correct, otherwise one may get misleading results.

This article describes a very powerful quantitative model which, if used properly, can be an excellent tool for corporate strategic planning. This quantitative model has been developed by Charnes and Cooper[1] as an extension of linear programming called “goal programming”. The article will show how powerful this model is and also that if the model is structured incorrectly it will produce misleading results. What follows is a brief presentation of goal programming.

Goal programming

Goal programming (GP) is an extension of linear programming (LP). Many readers are familiar with the basic assumption and concepts related to LP, a method of optimization utilized successfully over four decades. It deals with solving management problems of the optimization of a single objective, subject to several linear constraints. Although LP is used widely in decision-making processes, it has a major limitation which restricts the users of the technique to narrowing their problems to a single objective function.

In real life, decision situations frequently are characterized by multiple and conflicting objectives. For instance, today business firms operate in a competitive environment which is heavily influenced by external factors including regulators, competitors, suppliers, customers, shareholders and the public. These factors, in turn, make profit maximization no longer the sole objective for management.

Besides profit maximization, management now has many other objectives to attain, including improved market share, improved relationships with the public, production of a quality product, minimized employee unrest, minimized pollution and the realization of a certain rate of return on investment. Many of these objectives may conflict and so compete for scarce resources.

If the existence of multiple conflicting objectives is recognized, it follows that an ideal decision technique is one that is able to take them all into account. LP is an inadequate technique for solving problems of this type.

GP was developed to handle problems with multiple objectives. Used properly, this technique can aid decision makers in effective short- and long-range strategic planning.

The structure of GP model

The basic steps used in structuring an LP model are identical to those for GP. The major difference between the two is that the GP model does not optimize (maximize/minimize) the objective directly, as in the case of LP. Instead, it attempts to minimize the deviations between the desired goals and the realized results. These goals are prioritized in a hierarchy of importance.

To quantify this prioritization, each goal is expressed as an equation and a deviational variable(s) assigned to it. Deviational variables can be positive or negative. A positive deviational variable (d^+) represents overachievement of the goal. A negative deviational variable (d^-) represents underachievement of the goal. If the desire is not to underachieve the goal, d^- should be driven to zero. To the contrary, if d^+ is driven to zero, the overachievement of the goal will not be realized.

It may be noted that the structure of a GP model involves two types of constraint: system and goal constraints. The system constraints are those which are more restrictive in nature and have to be satisfied before the goal constraints, as they represent the existing capabilities, rather than what we would like to achieve.

From the above discussion, it can be deduced that deviational variables are mutually exclusive. This relationship is mathematically expressed as:

$$d_j^+ * d_j^- = 0.$$

The steps needed to structure a GP model are threefold:

- (1) Goals are identified and expressed as constraints.
- (2) Goals are analysed to determine the correct deviational variables needed for them, d_j^- , d_j^+ , or both.
- (3) A hierarchy of importance among goals is established by assigning to each of them a pre-emptive priority factor, P_j . These pre-emptive priority factors reflect the hierarchical relationships in such a way that P_1 represents the highest priority, P_2 the second highest, and so on. In other words, the P s indicate a simple ordinal ordering of the goals.

Once the above steps are completed, the problem can be quantified as a GP model as follows:

$$\text{Minimize: } \sum_{j=1}^n P_j (d_j^- + d_j^+), \text{ where } j = 1, 2, 3, \dots, n$$

$$\text{Subject to: } \sum_{i=1}^n (a_{ij} x_j) - d_i^+ + d_i^- = b_i, \text{ where } i = 1, 2, \dots, m.$$

Having briefly introduced the readers to the basic requirements for structuring a GP model, we can consider how the improper structure of the model leads to misleading results. Two illustrative examples are used demonstrate the above argument.

Example 1

The XYZ Company manufactures and markets two types of colour television set. Type X_1 contributes \$10 to profit, while type X_2 contributes \$12 to profit. The delivery of these two television sets to distributors is limited by manufacturing and assembly capacities of 120 and 64 hours, respectively. Television type X_1 requires six hours of manufacturing capacity while type X_2 requires 12 hours; in the assembly capacity, television type X_1 requires eight hours and type X_2 requires four hours.

If the president of the XYZ Company has only one objective, such as the maximization of profit, the problem may be formulated in an LP form as follows:

$$\begin{aligned} \text{Maximize: } & 10X + 12Y = Z \\ \text{Subject to: } & \begin{cases} 6X + 12Y \leq 120 & (1) \\ 8X + 4Y \leq 64 & (2) \end{cases} \\ \text{and} & X, Y \geq 0 \end{aligned}$$

Assume that the president of the XYZ Company has two prioritized goals to achieve:

- (1) To realize at least \$2,000. This strategy is considered to be the primary goal (P_1).
- (2) To maximize the utilization of the manufacturing and assembly capacities. This strategy is considered to be the secondary goal (P_2).

Here, the president of XYZ Company would like to see whether or not the given resources are sufficient to achieve his goals (this is the short-term strategy). If, however, the available resources cannot attain the specified goals, the president of the company would like to determine what strategy should be followed to realize these two goals simultaneously (this is the long-term strategy).

The answer to the above two questions (short-term strategy and long-term strategy) requires two different structures of the GP model as follows:

- (1) The short-term GP model

$$\begin{aligned} \text{Minimize: } & P_1(d_1^-) + 0d_1^+ + P_2(d_2^- + d_3^-) \\ \text{Subject to: } & \begin{cases} 10X_1 + 12X_2 + d_1^- - d_1^+ = 2,000 & (3) \\ 6X_1 + 12X_2 + d_2^- & = 120 & (4) \\ 8X_1 + 4X_2 + d_3^- & = 64 & (5) \end{cases} \\ \text{and } & X_1, X_2, \text{ and } d_1^-, \dots, d_3^-, d_1^+ \geq 0. \end{aligned}$$

- (2) The long-term GP model

$$\begin{aligned} \text{Minimize: } & P_1(d_1^-) + 0d_1^+ + P_2(d_2^- + d_3^-) + 0d_2^+ + 0d_3^+ \\ \text{Subject to: } & \begin{cases} 10X_1 + 12X_2 + d_1^- - d_1^+ & = 2,000 & (6) \\ 6X_1 + 12X_2 + d_2^- - d_2^+ & = 120 & (7) \\ 8X_1 + 4X_2 + d_3^- - d_3^+ & = 64 & (8) \end{cases} \\ \text{and } & X_1, X_2, \text{ and } d_1^-, \dots, d_3^-, d_1^+, d_2^+, d_3^+ \geq 0. \end{aligned}$$

Table AI of the Appendix shows the input and output of the short-term solution obtained by software developed by Dennis and Dennis[2].

The analysis of the objective function section of Table AI shows that P_1 , the first priority goal, is underachieved by 1,864, while P_2 is fully achieved. Another way to verify this point is by taking a glance at the variables in the solution stub of Table AI. It can be noticed that the deviational variable d_1^- , has a value of 1,864. One can also notice that the d_2^- and d_3^- variables are not in the solution stub, which means that their values are zero.

Moreover, the solution stub shows that the best product mix in the short term is to produce four units of X_1 and eight units of X_2 . This will generate a profit of \$136 (2,000 – 1,864).

Important remarks

From the foregoing, one may conclude that GP is not, as some people may think, a panacea that achieves goals one by one according to the priorities assigned to them. The proof of this argument is demonstrated in Table AI which shows that the P_1 goal (the higher in importance) is underachieved, while the P_2 (the lower in importance) is fully achieved. The reason for this is that the full utilization of the available resources (120 and 64 hours of the manufacturing and assembly capacities), respectively, cannot generate the \$2,000 profit set by management. That is why this answer is considered a short-term answer since resources are limited and cannot be expanded except in the long term.

If management needs to know from the beginning the amount of resource(s) needed to generate the minimum \$2,000 profit, it is necessary to restructure the short-term GP to a long-term one. This can be done by relaxing the manufacturing and the assembly capacities simultaneously. This relaxation can be accomplished by inserting in equations (4) and (5) of the short-term model the overachievement variables d_2^+ and d_3^+ , respectively. These two variables should also be assigned a zero pre-emptive priority factor in the objective function. Such a procedure creates the long-term GP model represented by equations (6)-(8).

Table AII shows the answer to the long-term model. The analysis of the objective function of Table AII indicates that both P_1 and P_2 goals (first and second priority goals) are fully achieved. The solution stub of the final tableau of Table AII indicates that the XYZ Company should increase the manufacturing and assembly capacities by 1,880 and 602.667 hours, respectively. In other words, in the long term, the manufacturing capacity should be expanded from 120 hours to 2,000 hours, while the assembly capacity should be increased from 64 hours to 666.667 hours in order to generate the required minimum \$2,000 profit. One can also notice that the product mix in the long term is totally different from that of the short term. The product mix for the long term is to produce 166.667 units of product X_2 only while the product mix for the short term is to produce four units of X_1 and eight units of X_2 .

The above presentation shows that an incorrect structure of a GP model definitely leads to a misleading solution. For this reason, it is important when structuring a GP model to be cautious about which deviational variable(s) is needed to achieve a certain goal.

This presentation shows also that decision makers should be careful in determining which deviational variable(s) needs to be minimized. For instance, if the objective is to minimize the underachievement variable (d_1^-), then this variable only should appear in the constraint that represents the objective in question. In other words, this is the variable that will be assigned a pre-emptive

priority factor P_j in the objective function. The other overachievement variable (d_1^+) should not be inserted in the constraint at all, since such an insertion will lift any restriction on the constraint in question.

Unfortunately, most management science books which discuss the GP model show goal constraints with d_1^- and d_1^+ simultaneously, even though the objective is to minimize the underachievement variable (d_1^-). Such a presentation can be misleading as shown already.

Another situation in which the improper structure of the GP model leads to a wrong result occurs when one has two goals: one, for example, to minimize idle capacity and the other to allow this capacity a limited amount of overtime. The example which follows explains this point.

Example 2

Let us assume that the president of the above XYZ Company has three prioritized goals to achieve:

- (1) To realize at least \$2,000 (P_1).
- (2) To maximize the utilization of the manufacturing and assembly capacities (P_2).
- (3) To allow a maximum of five hours overtime to each of the manufacturing and assembly capacities (P_3).

The first time users of the GP model will formulate this problem as follows:

Minimize: $P_1 (d_1^- + 0d_1^+) + P_2(d_2^- + d_3^-) + P_3(d_4^+ + d_5^+)$

$$\text{Subject to: } \begin{cases} 10 X_1 + 12 X_2 + d_1^- - d_1^+ = 2,000 & (9) \\ 6 X_1 + 12 X_2 + d_2^- = 120 & (10) \\ 8 X_1 + 4 X_2 + d_3^- = 64 & (11) \\ 6 X_1 + 12 X_2 + d_4^- - d_4^+ = 125 & (12) \\ 8 X_1 + 4 X_2 + d_5^- + d_5^+ = 69 & (13) \end{cases}$$

$$X_1, X_2 \text{ and } d_1^-, \dots, d_5^-, d_1^+, d_4^+, d_5^+ \geq 0.$$

Note that equations (10) and (11) have the underachievement variable d_2^- and d_3^- only since the second goal, P_2 , is to maximize the utilization of the manufacturing and assembly capacities. Therefore, assigning P_2 to d_2^- and d_3^- in the objective function will derive d_2^- and d_3^- to zero.

With regard to the third goal, P_3 , the objective is to allow a maximum of five hours' overtime to the manufacturing and assembly capacities, respectively. In other words, we do not want the manufacturing capacity to exceed 125 hours. Similarly, the assembly capacity should not exceed 69 hours. This is done by assigning P_3 to d_4^+ and d_5^+ in the objective function since d_4^+ and d_5^+ represent any hour above the permissible five hours' overtime allowed to each capacity.

Table AIII shows the computer solution to the above GP model. If we compare the answer of Table AIII with that of Table AI, we notice that both are identical. Both show that the best product mix is to produce four units of X_1 and eight units of X_2 . Both also show that goal, P_1 is underachieved by 1,864.

One may ask how it is that we get the same answer even though we allowed each capacity five hours' overtime? The answer is that although we described each goal correctly, it is very important to see that all goals are logically structured. For instance, equations (10) and (11) represent goal P_2 , which is the minimization of idle capacities; that is why we were interested in deriving d_2^- and d_3^- to zero. However, goal P_3 , which allows each capacity a maximum five hours overtime cannot be accomplished unless we relax both the manufacturing and assembly capacities simultaneously. In other words, equations (10) and (11) in the above model should be written as follows:

$$6X_1 + 12X_2 + d_2^- - d_2^+ = 120$$

$$8X_1 + 4X_2 + d_3^- - d_3^+ = 64.$$

We need to add to these equations the overachievement variables d_2^+ and d_3^+ , and we should not put any restriction on these variables in the objective function. In other words, these variables should have no priorities in the objective function as follows:

$$\text{Minimize: } P_1(d_1^- + 0d_1^+) + P_2(d_2^- + d_3^-) + 0d_2^+ + 0d_3^+ + P_3(d_4^- + d_5^-).$$

Without being alert to such a manipulation, one will get a biased answer without being able to know the reason. This is because the majority of management science books do not explain this point clearly.

Concluding remarks

The foregoing demonstrates the advantages of GP in improving the quality of the decision-making process of any organization. Some of the underlying assumptions may be found to be relatively unrealistic; for example, many relationships are really curvilinear. Solutions require implementations which may be difficult or impractical. Managers must strike a balance between personal, institutional and environmental goals to attain satisfactory results.

Yet, a strategy generating a marginal difference in efficiency may give one company the edge on a competitor, or may provide an increment of cost saving sufficient to stabilize customer service fees, or may provide sufficient marginal profits to enable it to survive and prosper. GP is applicable to many settings and types of decision, and from a theoretical perspective is worthy of a trial application.

One may also note that, in order to derive a meaningful solution, decision makers must give considerable attention to the formulation of the GP model. They must be precise and accurate in determining overachievement and underachievement variables necessary for each goal and be able to understand

the harmony among all specified goals. Lack of accuracy can yield a wrong result.

Unfortunately, the majority of management science books do not explain clearly the proper way of structuring the GP model. For instance, several books which are cited under further reading, represent GP examples that give the readers the impression that every goal should include d^- and d^+ variables even though the objective is to minimize the underachievement of the goal.

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Appendix. Goal programming

Short-term solution

Minimize: $0X_1 + 0X_2 + P_1d_1^- + P_2d_2^- + P_2d_3^- + 0d_1^+$

$$\text{Subject to: } \begin{cases} 10X_1 + 12X_2 + d_1^- - d_1^+ = 2,000 \\ 6X_1 + 12X_2 + d_2^- = 120 \\ 8X_1 + 4X_2 + d_3^- = 64 \end{cases}$$

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Table AI.
Solution stub of the
final tableau

Variables	RHS
d_1^-	1,864
X_2	8
X_1	4
$c_j - z_j P_2$	0
$c_j - z_j P_1$	1,864

Analysis of the objective function

Priority Non-achievement

P_1 1,864

P_2 0

Long-term solution

Minimize: $0X_1 + 0X_2 + P_1d_1^- + P_2d_2^- + P_2d_3^- + 0d_1^+ + 0d_2^+ + 0d_3^+$

$$\text{Subject to: } \begin{cases} 10X_1 + 12X_2 + d_1^- - d_1^+ = 2,000 \\ 6X_1 + 12X_2 + d_2^- - d_2^+ = 120 \\ 8X_1 + 4X_2 + d_3^- - d_3^+ = 64 \end{cases}$$

Table AII.
Solution stub of the
final tableau

Variables	RHS
d_3^+	602.66
X_2	166.667
d_2^+	1,880
$c_j - z_j P_2$	0
$c_j - z_j P_1$	0

Analysis of the objective function

Priority Non-achievement

P_1 0

P_2 0

Allowing overtime solution

Minimize: $0X_1 + 0X_2 + P_1d_1^- + P_2d_2^- + P_2d_3^- + P_3d_4^- + P_3d_5^- + 0d_1^+ + 0d_2^+ + 0d_3^+$

$$\text{Subject to: } \begin{cases} 10X_1 + 12X_2 + d_1^- - d_1^+ = 2,000 \\ 6X_1 + 12X_2 + d_2^- = 120 \\ 8X_1 + 4X_2 + d_3^- = 64 \\ 6X_1 + 12X_2 + d_4^- - d_4^+ = 125 \\ 8X_1 + 4X_2 + d_5^- - d_5^+ = 69 \end{cases}$$

Variables	RHS
d_3^-	1,864
X_2	8
X_1	4
d_4^-	5
d_5^-	5

Table AIII.
Allowing overtime
solution

Analysis of the objective function

Priority	Non-achievement
P_1	1,864
P_2	0
P_3	10